Statistical Mechanics of Fermi Acceleration

Austin Hill – Western Washington University

Mentor: Nasser Barghouty, ZP12

Abstract

Cosmic rays are a form of high-energy radiation moving at relativistic speeds consisting primarily of protons, the nuclei of elements (primarily hydrogen and helium, but also heavier elements), alongside a small fraction of electrons. The Fermi Acceleration mechanism is one that posits that cosmic rays can be accelerated to high energies by repeated scattering through magnetic clouds that also inhabit the interstellar regions of the galaxy. In my research, work is being done to create a more stable mathematical grounding of these hypotheses as I create and analyze statistical mechanics of various mechanisms incorporated within.

Goals

The ultimate goal is to be able to mathematically account for the existence of cosmic rays. In order to do so, specific characteristics must be accounted for in any given model.

1. The energy spectrum of cosmic rays takes the form of a power law,
$$dN(E) \propto E^{-x} dE$$
where $x = 2.5 - 2.7$ at energies $\sim 1 - 10^3\ GeV$

2. The acceleration of cosmic rays to energy levels on the magnitude of $E \sim 10^{20}\ eV, or$ approximately 16 joules.

Methodology

The primary methodology of research is in the realm of statistical mechanics – the use of probability theory to determine the average behavior of a mechanical system. Additionally, a combination of theoretical physics and applied mathematics has been used.

References

1. Michael W. Friedlander, *Cosmic Rays*. Harvard University Press, Cambridge, MA, 1989.
2. Christopher Jarzynski, *Studies in Chaotic Adiabatic Dynamics*. Physics Department, UC Berkeley, 1994.
3. Malcolm S. Longair, *High Energy Astrophysics*. The Press Syndicate of the University of Cambridge, UK, Volume 2, 2nd Edition, 2002.
4. Pasquale Blasi, *Particle Acceleration*, INAF/Arcetri Astrophysical Observatory, Lecture 2, Summer 2012.

Second Order Fermi Acceleration

Before the analysis of the statistical mechanics, I began by rigorously analyzing the derivation of Fermi's second order acceleration method. The principal of the method is that charged particles are statistically able to gain energy through repeated reflections with electromagnetic clouds in interstellar space.

Derivation

During derivation, the assumption is made that the velocity of the particles is approximately equal to the speed of light; as a result, operations must be made utilizing relativistic motion equations.

Assume the existence of a particle with energy E and velocity v that exists within the reference frame of an electromagnetic cloud moving with velocity V

In this reference frame, particle energy is: $E' = \gamma E + \mathrm{B}\gamma p\mu$

Particle relativistic three-momentum, $p'_x = \beta\gamma E + \gamma p\mu$

With these assumptions, we can determine the final energy in the lab frame to be:

$$E'' = \gamma E' + \beta\gamma p'_x = \gamma^2 E\left(1 + \beta^2 + 2\beta\mu\left(\frac{p}{E}\right)\right)$$

Based on the relativistic kinematics, it is known that:

$$\frac{p}{E} = \frac{m_0 v\gamma}{m_0\gamma} = v, \qquad the\ dimensionless\ velocity$$

Simplifying these two equations together,

$$\frac{E'' - E}{E} = \gamma^2(1 + 2\beta v\mu + \beta^2) - 1$$

After taking the limit of non-relativistic clouds ($g \to 1$),

$$\frac{E'' - E}{E} \approx 2\beta^2 + 2\beta v\mu$$

The scattering probability along a direction is proportional to the relative velocity in that direction:

$$P(\mu) = Av_{rel} = \frac{A(\beta\mu + v)}{1 + v\beta\mu} \to_{v\to 1}$$
$$\approx A(1 + \beta\mu)$$
$$\left\langle\frac{\Delta E}{E}\right\rangle = \int_{-1}^{1} P(\mu)d\mu(2\beta^2 + 2\beta\mu)$$
$$= {}^{8}/_{3}\,\beta^2$$
$$= {}^{8}/_{3}\left(\frac{V}{C}\right)^2$$

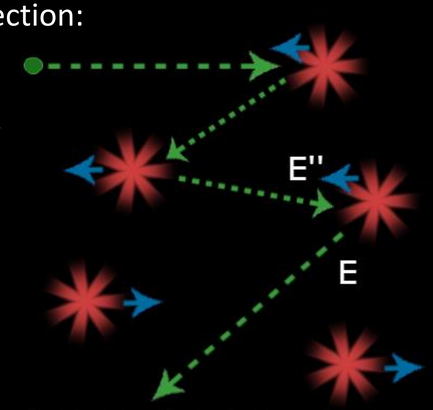

Fig. 1 Depiction of Particle Collision and Reflections

Statistical Mechanic: Particle Collision Probability

Goal: Display a proportionality between rate of particle collisions and relative velocity

$P(t) = the\ probability\ a\ particle\ survives\ time\ t$

$w\ dt = the\ probability\ a\ particle\ suffers\ a\ collision\ between\ time\ t\ and\ t + dt$

$\to P(t + dt) = P(t)(1 - w\ dt)$
$\to P(t) = e^{-wt}$

$\tau = \bar{t} = \frac{1}{w}$, the average time betwen collisions

$l(v) = v\tau(v)$, $where\ l\ is\ the\ mean\ free\ path$

By analyzing the total scattering cross section of the particles, it can be shown that

$w = \tau^{-1} = \bar{V}\sigma_0 n, \qquad where\ V = v_1 - v_2$

Thus, $w \propto V_{rev}$

Future Work

This work has been done with the goal of creating a link between Second Order Fermi Acceleration and Chaotic Adiabatic Dynamics, the 1994 Ph.D. topic of Christopher Jarzynski of UC Berkeley. Chaotic Adiabatic Dynamics (CAD) refers to the study of adiabatically closed systems exhibiting chaotic evolution under slowly time-dependent motion equations.

Acknowledgements

Austin Hill thanks Dr. Nasser Barghouty for his guidance, mentorship, and intellectual stimulation throughout his role as Principal Investigator of this project, alongside the MSFC and NSSTC as a whole. This work was funded by both the Washington Space Grant and Marshall Space Flight Center.